A Dynamic Event-Triggered Approach to Recursive Filtering for Complex Networks With Switching Topologies Subject to Random Sensor Failures

Qi Li, Zidong Wang, Nan Li, and Weiguo Sheng

***Abstract*— This article deals with the recursive filtering issue for a class of nonlinear complex networks (CNs) with switching topologies, random sensor failures and dynamic event-triggered mechanisms. A Markov chain is utilized to characterize the switching behavior of the network topology. The phenomenon of sensor failures occurs in a random way governed by a set of stochastic variables obeying certain probability distributions. In order to save communication cost, a dynamic event-triggered transmission protocol is introduced into the transmission channel from the sensors to the recursive filters. The objective of the addressed problem is to design a set of dynamic event-triggered filters for the underlying CN with a certain guaranteed upper bound (on the filtering error covariance) that is then locally minimized. By employing the induction method, an upper bound is first obtained on the filtering error covariance and subsequently minimized by properly designing the filter parameters. Finally, a simulation example is provided to demonstrate the effectiveness of the proposed filtering scheme.**

***Index Terms*— Complex networks (CNs), dynamic event-triggered mechanisms (ETMs), recursive filtering, sensor failures, switching topologies.**

I. Introduction

Complex networks (CNs) are typically made up of a great number of interconnected nodes under a certain topology structure, where each node can be regarded as a dynamical subsystem having its own physical meaning. In the real world, CNs are capable of describing various types of dynamical networks including, but are not limited to, information networks, social networks, biological networks, technological networks, and electrical power grids (see [1]–[3], [5], [12], [22], [24], [30], [32], and the reference therein). For decades, the dynamics analysis issues of CNs have received an ever-increasing research interest and there have been a rich body of excellent results available in the literature (see [10], [40], [45], [46] on the stability issue; [6], [7], [33], [47], [48] on the synchronization issue; and [11], [41], [42] for the pinning control issue).

For CNs, it is of vital importance to acquire the accurate state information of the network in order to achieve certain requirements such as synchronization and consensus. Due to the large scale of CNs, it is often unbearably expensive to fully access all the network states, and an alternative yet practical way is to estimate the network states based on the available measurements collected from the deployed sensors. As such, the filtering problems for CNs have recently attracted much research attention (see [15], [17], [20], [21], [31]). It is notable that, in most available literature on the filtering problems for CNs, the sensors have been implicitly assumed to be free of failures. In reality, however, the phenomenon of sensor failures is often encountered owing primarily to unavoidable sensor aging and component constraints, and the occurrence of such a phenomenon may appear in a probabilistic way because of the random changes in working conditions. Accordingly, some initial results have been published on the filtering issues for systems undergoing possible sensor failures (see [18], [28]).

In practice, it is often the case that the network communication topologies change over time because of a variety of reasons including link failures, channel blocking, and recreation of the network. Recently, the dynamics analysis problem for CNs with time-varying topologies has become a hot topic of research with some inspiring results reported in the literature (see [8], [17], [37], [39], and the reference therein). For example, a set of nonfragile state estimators has been designed in [39] for CNs with switching topologies governed by a sojourn-probability-based random process. In [8], the H_∞ state estimation problem has been dealt with for a class of CNs whose topologies are modeled by resorting to a Markov chain. Nevertheless, the variance-constrained recursive filtering issue for CNs with time-varying topologies has not yet gained adequate research efforts, and this gives rise to one motivation of the current investigation.

On another research front, in the past decade, there has been a growing research interest in the filtering/control issues based on the event-triggered mechanism (ETM) for the purpose of saving communication energy. The core idea of the ETM is to execute certain tasks only when the predefined triggering rule is satisfied. Roughly speaking, ETMs can be classified as static ETMs and dynamic ETMs, where the former has been widely investigated in the existing literature (see [16], [18], [19], [34], [35], [43], [44]). It is worth mentioning that, compared to the static ETMs with fixed threshold parameter, the dynamic ETMs are capable of adjusting the threshold parameters in a dynamical way via introducing a dynamic variable, thereby generating fewer triggers and further improving the resource utilization. As such, the filtering/control issues under the dynamic ETMs have recently begun to stir quite a lot research attention (see [9], [13], [14], [36] for some representative results). Nevertheless, when it comes to CNs, the corresponding recursive filtering problem under dynamic ETMs has not been considered yet, and another motivation of this article is therefore to shorten such a gap.

According to the aforementioned discussion, it is theoretically important and practically significant to develop a recursive filtering algorithm for CNs subject to switching topologies and random sensor failures under dynamic ETMs. To carry out the proposed research, three main difficulties have emerged as follows: 1) the examination on the impacts from dynamic ETMs, random sensor failures and switching topologies on the performance of the designed recursive filter; 2) the development of effective methods for CNs to find an

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Q. Li and W. Sheng are with the Institute of Service Engineering, Hangzhou Normal University, Hangzhou 311121, China (e-mail: weiguouk@hotmail.com).

Z. Wang is with the Department of Computer Science, Brunel University London, London UB8 3PH, U.K. (e-mail: zidong.wang@brunel.ac.uk).

N. Li is with the School of Information Science and Technology, Donghua University, Shanghai 200051, China.

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upper bound on the filtering error covariance; and 3) the parameterization of desired recursive filter gains that minimize the obtained upper bound. In this article, we endeavor to overcome the identified three difficulties by establishing a unified recursive approach to deal with the addressed filtering problem.

In this article, we are concerned with the recursive filtering issue for a class of nonlinear CNs with switching topologies, random sensor failures, and dynamic ETMs, where the network topology switches according to a Markov chain, the sensor failures occur randomly with certain probability distribution, and the ETM governs the signal transmissions in a dynamical way. Our ultimate goal is to design a set of filters that guarantee a locally minimized upper bound on the filtering error covariance. The main contributions of this article are highlighted as follows: 1) the recursive filtering problem is new for the addressed nonlinear CNs subject to Markovian switching topologies and random sensor failures; 2) the dynamic ETM is, for the first time, taken into account on the design of recursive filters for CNs for the purpose of saving energy; and 3) the desired filters are determined by solving a set of recursive equations, thereby facilitating online applications.

The subsequent part of this article is organized as follows. In Section II, the recursive filtering problem is formulated for the CNs with dynamic ETMs. Section III derives the main results for designing the desired filter. In Section IV, a simulation example is provided to illustrate the effectiveness of the proposed filtering algorithm. Finally, the conclusion is drawn in Section V.

Notation: In this article, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The superscript "T" refers to matrix transposition. I denotes the identity matrix. $\mathbb{E}(x)$ means the expectation of the stochastic variable x, and $\Pr\{\cdot\}$ is the occurrence probability of event "$\cdot$"; $\operatorname{tr}\{A\}$ denotes the trace of a matrix A. For real symmetric matrices X and Y, $X \geq Y$ $(X > Y)$ indicates that $X - Y$ is positive semi-definite (positive definite). $\|\cdot\|$ is the Euclidean norm.

II. Problem Formulation

Consider the following class of CNs consisting of n coupled nodes defined on a finite horizon $k \in [0, N]$:

$$x_{i,k+1} = f_k(x_{i,k}) + \sum_{j=1}^{n} w_{ij,\pi_k}\Gamma x_{j,k} + B_{i,k}w_k \tag{1}$$

for $i = 1, 2, \ldots, n$, where $x_{i,k} \in \mathbb{R}^{n_x}$ is the state vector of the ith node. $W_{\pi_k} = (w_{ij,\pi_k})_{n\times n}$ is the coupled configuration matrix of the network with $w_{ij,\pi_k} \geq 0$ $(i \neq j)$ but not all zeros, and $W_{\pi_k} = W_{\pi_k}^T$ satisfies $w_{ii,\pi_k} = -\sum_{j=1,j\neq i}^{n} w_{ij,\pi_k}$. $\Gamma = \operatorname{diag}\{r_1, r_2, \ldots, r_{n_x}\}$ is an inner-coupling matrix. $B_{i,k}$ is a known time-varying matrix. $w_k \in \mathbb{R}^{n_w}$ is the process noise. The initial value $x_{i,0}$ has the mean $\bar{x}_{i,0}$ and covariance $P_{i,0} > 0$. The random variable π_k is a homogeneous Markov chain taking values in the finite set $S = \{1, 2, \ldots, s\}$. The transition probability matrix is $\Lambda \triangleq [p_{mu}]_{s\times s}$ with $p_{mu} \triangleq \Pr\{\pi_{k+1} = u|\pi_k = m\}$, $0 \leq p_{mu} \leq 1$, and $\sum_{u=1}^{s} p_{mu} = 1$.

Assumption 1 [23]: The nonlinear function $f_k(\cdot)$ in (1) satisfies $f_k(0) = 0$ and

$$\|f_k(x) - f_k(y) - E_k(x - y)\| \leq \alpha_k\|x - y\| \tag{2}$$

for all $x, y \in \mathbb{R}^{n_x}$, where E_k is a known time-varying matrix and α_k is a known nonnegative scalar.

Remark 1: The CN introduced in this article can be used to model rich dynamical behaviors of a variety of dynamical networks including recurrent neural networks (RNNs) as a typical example. For RNNS, the neurons are highly interconnected dynamical units/nodes, where the connections between neurons form a directed graph along a temporal sequence [4]. It is known from [26] that neuronal activity gives rise to dynamic patterns and processes, and anatomical/structural connections create a structural skeleton for a dynamic regime conducive to flexible and robust neural computation. Note that RNNs have been extensively applied in various domains such as machine translation, time series prediction and speech recognition. Also, techniques/theories for analyzing CNs have begun to be employed to study deep learning systems (e.g., deep belief networks) in order to gain insights into the structural and functional properties of the computational graph resulting from the learning process [27], [29].

Remark 2: As is well known, it is usually difficult to deal with the filtering problem for a general nonlinear system without any constraints. For the convenience of analysis and synthesis, we make an assumption on the nonlinear function $f_k(\cdot)$ in (1). Such an assumption has been widely adopted in the existing literature (see [23]). Furthermore, note that when $E_k = 0$, (2) reduces to the traditional Lipschitz condition $\|f_k(x) - f_k(y)\| \leq \alpha_k\|x - y\|$. Therefore, the nonlinearity description in (2) is quite general that covers the Lipschitz condition.

For node i $(1 \leq i \leq n)$, the measurement output with random sensor failures is described as follows:

$$y_{i,k} = \gamma_{i,k}C_{i,k}x_{i,k} + D_{i,k}v_k \tag{3}$$

where $y_{i,k} \in \mathbb{R}^{n_y}$ is the measurement output of the ith node. $v_k \in \mathbb{R}^{n_v}$ is the measurement noise. $C_{i,k}$ and $D_{i,k}$ are known matrices. The random variable $\gamma_{i,k}$, which governs the phenomenon of the random sensor failures, takes values on the interval $[0, 1]$ according to a certain probabilistic density function with mean $\bar{\gamma}_i$ and variance $\tilde{\gamma}_i$.

Remark 3: The measurement model described by (3) is capable of characterizing the phenomenon of probabilistic sensor failures by resorting to the random variable $\gamma_{i,k}$, where $\gamma_{i,k} = 1$ means that sensor i works normally, while $\gamma_{i,k} \neq 1$ means that the sensor i fails with a certain degree. Note that this measurement model has been used in existing literature (see [18]).

Assumption 2: The process noise w_k and the measurement noise v_k are two uncorrelated zero-mean Gaussian white-noise sequences with covariances $R_k > 0$ and $Q_k > 0$, respectively.

Assumption 3: Throughout this article, all the random variables, namely, w_k, v_k, $x_{i,0}$, $\gamma_{i,k}$, and π_k, are mutually independent.

In order to save energy, for each node, a dynamic ETM is employed to decide whether the current measurement should be transmitted to the filter or not. Denote the triggering instant sequence by $0 \leq t_0^i < t_1^i < \cdots < t_l^i < \cdots$, where t_{l+1}^i is determined based on the following rule:

$$t_{l+1}^i = \min\left\{k \in [0, N]|k > t_l^i,\ \frac{1}{\theta_i}\eta_{i,k} + \sigma_i - \|\varepsilon_{i,k}\| \leq 0\right\}. \tag{4}$$

Here, σ_i and θ_i are given positive scalars, $\varepsilon_{i,k} \triangleq y_{i,k} - y_{i,t_l^i}$, y_{i,t_l^i} denotes the transmitted measurement at latest event time, and $\eta_{i,k}$ is an internal dynamic variable satisfying

$$\eta_{i,k+1} = \lambda_i\eta_{i,k} + \sigma_i - \|\varepsilon_{i,k}\|, \quad \eta_{i,0} = \eta_0^i \tag{5}$$

where λ_i is a given positive scalar and $\eta_0^i \geq 0$ is the given initial condition.

Remark 4: When $\theta_i \to +\infty$, (4) reduces to $t_{l+1}^i = \min\{k \in [0, N]|k > t_l^i, \sigma_i - \|\varepsilon_{i,k}\| \leq 0\}$, which is actually the traditional static ETM used in [19]. Therefore, the proposed dynamic event-triggered method includes the static one as a special case.

By means of the introduced dynamic event-triggered measurements, we construct the following filter for node i $(1 \le i \le n)$:

$$\begin{aligned} \hat{x}_{i,k+1|k} &= f_k(\hat{x}_{i,k|k}) + \sum_{j=1}^{n} w_{ij,\pi_k}\Gamma\hat{x}_{j,k|k} \\ \hat{x}_{i,k+1|k+1} &= \hat{x}_{i,k+1|k} + K_{i,k+1}(y_{i,t_l^i} - \bar{\gamma}_i C_{i,k+1}\hat{x}_{i,k+1|k}) \end{aligned} \tag{6}$$

for $k+1 \in [t_l^i, t_{l+1}^i)$ $(l \ge 0)$, where $\hat{x}_{i,k+1|k}$ and $\hat{x}_{i,k|k}$ are, respectively, the one-step prediction and the updated estimate of the state $x_{i,k}$ at time-instant k. Here, $K_{i,k+1}$ is the filter gain to be designed. The initial value is set as $\hat{x}_{i,0|0} = \bar{x}_{i,0}$.

Recalling the definition of $\varepsilon_{i,k}$, (6) can be rewritten as

$$\begin{aligned} \hat{x}_{i,k+1|k} &= f_k(\hat{x}_{i,k|k}) + \sum_{j=1}^{n} w_{ij,\pi_k}\Gamma\hat{x}_{j,k|k} \\ \hat{x}_{i,k+1|k+1} &= \hat{x}_{i,k+1|k} + K_{i,k+1} \\ &\quad \times(\gamma_{i,k+1}C_{i,k+1}x_{i,k+1} + D_{i,k+1}v_{k+1} - \varepsilon_{i,k+1} \\ &\quad - \bar{\gamma}_i C_{i,k+1}\hat{x}_{i,k+1|k}). \end{aligned} \tag{7}$$

Letting the prediction error be $e_{i,k+1|k} \triangleq x_{i,k+1} - \hat{x}_{i,k+1|k}$ and the filtering error be $e_{i,k+1|k+1} \triangleq x_{i,k+1} - \hat{x}_{i,k+1|k+1}$, we have from (1) and (7) that

$$\begin{aligned} e_{i,k+1|k} &= \tilde{f}_k(e_{i,k|k}) + \sum_{j=1}^{n} w_{ij,\pi_k}\Gamma e_{j,k|k} + B_{i,k}w_k \\ e_{i,k+1|k+1} &= (I - \bar{\gamma}_i K_{i,k+1}C_{i,k+1})e_{i,k+1|k} + K_{i,k+1}\varepsilon_{i,k+1} \\ &\quad - (\gamma_{i,k+1} - \bar{\gamma}_i)K_{i,k+1}C_{i,k+1}x_{i,k+1} \\ &\quad - K_{i,k+1}D_{i,k+1}v_{k+1} \end{aligned} \tag{8}$$

where $\tilde{f}_k(e_{i,k|k}) \triangleq f_k(x_{i,k}) - f_k(\hat{x}_{i,k|k})$.

In this article, our purpose is to design a dynamic event-triggered filter of form (6) for the CN (1) such that, in the presence of switching topologies and random sensor failures, the filtering error covariance

$$P_{i,k+1|k+1} \triangleq \mathbb{E}\{e_{i,k+1|k+1}e_{i,k+1|k+1}^T\}, \quad (1 \le i \le n)$$

has an upper bound and such a bound is then minimized at each time instant.

III. Main Results

In this section, an upper bound on the filtering error covariance is first derived and then minimized by designing appropriate filter parameters.

Before proceeding, the following lemmas are introduced to obtain our main results.

Lemma 1 [13]: For the introduced dynamic ETM (4), (5) with $\eta_0^i \ge 0$ $(1 \le i \le n)$, if the parameters λ_i and θ_i satisfy $\lambda_i\theta_i \ge 1$, then $\eta_{i,k}$ satisfies $\eta_{i,k} \ge 0$ for all $k \in [0, N]$.

Remark 5: From Lemma 1, it is seen that the introduced variable $\eta_{i,k}$ remains nonnegative for all $k \in [0, N]$. In this case, unlike what the static ETMs requires, the value of $\sigma_i - \|\varepsilon_{i,k}\|$ is no longer required to keep nonnegative always. Therefore, compared to the static ETMs, the triggering times under the proposed dynamic event-triggered method are reduced, thereby saving the network communication cost substantially.

Lemma 2 [19]: For all $k \in [0, N]$, let $H_k(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ be a matrix function, and X and Y be real-valued positive matrices. If $H_k(X) \le H_k(Y), \forall X \le Y$, then the solutions N_{k+1} and M_{k+1} to the following difference equations:

$$N_{k+1} = H_k(N_k), \quad M_{k+1} \le H_k(M_k), \; M_0 = N_0$$

satisfy $M_{k+1} \le N_{k+1}$.

Lemma 3: For two real-valued matrices M and N, the following inequality is true:

$$MN^T + M^T N \le \alpha MM^T + \alpha^{-1}NN^T$$

where α is an arbitrary positive scalar.

Lemma 4: For all $1 \le i \le n$, suppose that $\lambda_i\theta_i \ge 1$ holds and let the positive scalars $a_{i,k}$ and $b_{i,k}$ be given. If there exists a set of real-valued matrices $\bar{Y}_{i,k}$ satisfying

$$\begin{aligned} \bar{Y}_{i,k+1} &\triangleq \Omega_k^i(\bar{Y}_{i,k}) \\ &\triangleq \big((1+a_{i,k})(1+b_{i,k})\lambda_i^2 + (1+\theta_i) \times \big(1+a_{i,k}^{-1}\big)/\theta_i^2\big)\bar{Y}_{i,k} \\ &\quad + \big((1+a_{i,k})\big(1+b_{i,k}^{-1}\big) + \big(1+a_{i,k}^{-1}\big)\big(1+\theta_i^{-1}\big)\big)\sigma_i^2 \end{aligned} \tag{9}$$

with the initial condition $\bar{Y}_{i,0} = (\eta_0^i)^2$, then $\bar{Y}_{i,k}$ is an upper bound of $Y_{i,k} \triangleq \mathbb{E}\{\eta_{i,k}^2\}$, i.e., $Y_{i,k} \le \bar{Y}_{i,k}$.

Proof: By using Lemma 3, it follows from (4) that

$$\varepsilon_{i,k}^T\varepsilon_{i,k} \le \left(\frac{1}{\theta_i}\eta_{i,k} + \sigma_i\right)^2 \le (1+\theta_i)\eta_{i,k}^2/\theta_i^2 + \big(1+\theta_i{}^{-1}\big)\sigma_i^2. \tag{10}$$

From (5) and (10), we have

$$\begin{aligned} Y_{i,k+1} &= \mathbb{E}\{(\lambda_i\eta_{i,k} + \sigma_i - \|\varepsilon_{i,k}\|)^2\} \\ &\le (1+a_{i,k})(1+b_{i,k})\lambda_i^2\mathbb{E}\{\eta_{i,k}^2\} + (1+a_{i,k}) \\ &\quad \times \big(1+b_{i,k}^{-1}\big)\sigma_i^2 + \big(1+a_{i,k}^{-1}\big)\mathbb{E}\{\varepsilon_{i,k}^T\varepsilon_{i,k}\} \\ &\le \big((1+a_{i,k})(1+b_{i,k})\lambda_i^2 + (1+\theta_i)\big(1+a_{i,k}^{-1}\big)/\theta_i^2\big) \\ &\quad \times Y_{i,k} + \big((1+a_{i,k})\big(1+b_{i,k}^{-1}\big) + \big(1+a_{i,k}^{-1}\big) \\ &\quad \times \big(1+\theta_i^{-1}\big)\big)\sigma_i^2. \end{aligned} \tag{11}$$

Then, by using Lemma 2, we can easily obtain $Y_{i,k} \le \bar{Y}_{i,k}$, which ends the proof. ■

The following theorem provides an upper bound on the filtering error covariance at each time instant.

Theorem 1: For all $1 \le i \le n$, suppose that $\lambda_i\theta_i \ge 1$ and let the positive scalars $c_{i,k}$, $d_{i,k}$, $e_{i,k}$, $g_{i,k}$ and the gain matrix $K_{i,k}$ be given. Assume that there exist two sequences of real-valued matrices $\Pi_{i,k+1|k}$ and $\Pi_{i,k+1|k+1}$ satisfying

$$\begin{aligned} \Pi_{i,k+1|k} &\triangleq (1+c_{i,k})(1+d_{i,k})\alpha_k^2\mathrm{tr}\{\Pi_{i,k|k}\} + (1+c_{i,k}) \\ &\quad \times \big(1+d_{i,k}^{-1}\big)E_k\Pi_{i,k|k}E_k^T + \bar{w}_{i,\pi_k}\big(1+c_{i,k}^{-1}\big) \\ &\quad \times \sum_{j=1}^{n}|w_{ij,\pi_k}|\Gamma\Pi_{j,k|k}\Gamma^T + B_{i,k}R_kB_{i,k}^T \end{aligned} \tag{12}$$

and

$$\begin{aligned} &\Pi_{i,k+1|k+1} \\ &\triangleq (1+e_{i,k+1})(I - \bar{\gamma}_i K_{i,k+1}C_{i,k+1})\Pi_{i,k+1|k} \\ &\quad \times (I - \bar{\gamma}_i \times K_{i,k+1}C_{i,k+1})^T + \big(1+e_{i,k+1}^{-1}\big) \\ &\quad \times \big((1+\theta_i)\bar{Y}_{i,k+1}/\theta_i^2 + \big(1+\theta_i{}^{-1}\big)\sigma_i^2\big)K_{i,k+1}K_{i,k+1}^T \\ &\quad + \tilde{\gamma}_i\big(1+g_{i,k+1}\big)K_{i,k+1} \times C_{i,k+1}\Pi_{i,k+1|k}C_{i,k+1}^T K_{i,k+1}^T \\ &\quad + \tilde{\gamma}_i\big(1+g_{i,k+1}^{-1}\big)K_{i,k+1} \times C_{i,k+1}\hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^T C_{i,k+1}^T K_{i,k+1}^T \\ &\quad + \big(1-2\rho_{i,k+1}\big) \times K_{i,k+1}D_{i,k+1}Q_{k+1}D_{i,k+1}^T K_{i,k+1} \end{aligned} \tag{13}$$

with the initial condition $\Pi_{i,0|0} = P_{i,0|0} = P_{i,0}$, where $\bar{w}_{i,\pi_k} \triangleq \sum_{j=1}^{n}|w_{ij,\pi_k}|$ and $\rho_{i,k}$ is a binary variable defined as follows: at time-instant k, when the triggering condition (4) is satisfied, $\rho_{i,k} = 0$, otherwise $\rho_{i,k} = 1$. Then, $\Pi_{i,k+1|k+1}$ is an upper bound of the filtering error covariance $P_{i,k+1|k+1}$, i.e., $P_{i,k+1|k+1} \le \Pi_{i,k+1|k+1}$.

Proof: We will prove this theorem by using the mathematical induction method. Noting the initial condition $P_{i,0|0} = \Pi_{i,0|0}$, we assume that $P_{i,k|k} \leq \Pi_{i,k|k}$. Then, what we need to show is $P_{i,k+1|k+1} \leq \Pi_{i,k+1|k+1}$.

First, denoting $P_{i,k+1|k} \triangleq \mathbb{E}\{e_{i,k+1|k}e_{i,k+1|k}^T\}$ as the prediction error covariance, it is obtained from (8) that

$$\begin{aligned}P_{i,k+1|k} &= \mathbb{E}\{e_{i,k+1|k}e_{i,k+1|k}^T\}\\ &= \mathbb{E}\left\{\left(\tilde{f}_k(e_{i,k|k}) + \sum_{j=1}^{n} w_{ij,\pi_k}\Gamma e_{j,k|k} + B_{i,k}\times w_k\right)\right.\\ &\qquad\left.\times\left(\tilde{f}_k(e_{i,k|k}) + \sum_{j=1}^{n} w_{ij,\pi_k}\Gamma e_{j,k|k} + B_{i,k}w_k\right)^T\right\}.\end{aligned} \tag{14}$$

By using Lemma 3 and noting (3), one has

$$\begin{aligned}&P_{i,k+1|k}\\ &\leq \mathbb{E}\left\{(1+c_{i,k})(1+d_{i,k})\|\tilde{f}_k(e_{i,k|k}) - E_k e_{i,k|k}\|^2 I\right.\\ &\qquad + (1+c_{i,k})\left(1+d_{i,k}^{-1}\right)E_k e_{i,k|k}e_{i,k|k}^T E_k^T + \left(1+c_{i,k}^{-1}\right)\\ &\qquad\left.\times\sum_{j=1}^{n} w_{ij,\pi_k}\Gamma e_{j,k|k}\left(\sum_{l=1}^{n} w_{il,\pi_k}\Gamma e_{l,k|k}\right)^T\right\}\\ &\quad + B_{i,k}R_k B_{i,k}^T\\ &\leq (1+c_{i,k})(1+d_{i,k})\alpha_k^2 \mathrm{tr}\{P_{i,k|k}\}I + (1+c_{i,k})\\ &\quad\times\left(1+d_{i,k}^{-1}\right)E_k P_{i,k|k}E_k^T + \left(1+c_{i,k}^{-1}\right)\\ &\quad\times\mathbb{E}\left\{\sum_{j=1}^{n} w_{ij,\pi_k}\Gamma e_{j,k|k}\left(\sum_{l=1}^{n} w_{il,\pi_k}\Gamma e_{l,k|k}\right)^T\right\}\\ &\quad + B_{i,k}R_k B_{i,k}^T.\end{aligned} \tag{15}$$

For the term $\mathbb{E}\{\sum_{j=1}^{n} w_{ij,\pi_k}\Gamma e_{j,k|k}(\sum_{l=1}^{n} w_{il,\pi_k}\Gamma e_{l,k|k})^T\}$ in (15), one has

$$\begin{aligned}&\mathbb{E}\left\{\sum_{j=1}^{n} w_{ij,\pi_k}\Gamma e_{j,k|k}\left(\sum_{l=1}^{n} w_{il,\pi_k}\Gamma e_{l,k|k}\right)^T\right\}\\ &\quad\leq \frac{1}{2}\mathbb{E}\left\{\sum_{j=1}^{n}\sum_{l=1}^{n}|w_{ij,\pi_k}||w_{il,\pi_k}|\right.\\ &\qquad\left.\times\Gamma\left(e_{j,k|k}e_{j,k|k}^T + e_{l,k|k}e_{l,k|k}^T\right)\Gamma^T\right\}\\ &\quad= \bar{w}_{i,\pi_k}\sum_{j=1}^{n}|w_{ij,\pi_k}|\Gamma P_{j,k|k}\Gamma^T.\end{aligned} \tag{16}$$

Substituting (16) into (15) yields

$$\begin{aligned}P_{i,k+1|k} &\leq (1+c_{i,k})(1+d_{i,k})\alpha_k^2\mathrm{tr}\{P_{i,k|k}\}I + (1+c_{i,k})\\ &\quad\times\left(1+d_{i,k}^{-1}\right)E_k P_{i,k|k}E_k^T + \bar{w}_{i,\pi_k}\left(1+c_{i,k}^{-1}\right)\\ &\quad\times\sum_{j=1}^{n}|w_{ij,\pi_k}|\Gamma P_{j,k|k}\Gamma^T + B_{i,k}R_k B_{i,k}^T\end{aligned} \tag{17}$$

which implies

$$P_{i,k+1|k} \leq \Pi_{i,k+1|k}. \tag{18}$$

Next, we are ready to show that $P_{i,k+1|k+1} \leq \Pi_{i,k+1|k+1}$ is true. For node i $(1 \leq i \leq n)$, the filtering error covariance $P_{i,k+1|k+1}$ is calculated as follows:

$$\begin{aligned}&P_{i,k+1|k+1}\\ &\quad= \mathbb{E}\{((I - \bar{\gamma}_i K_{i,k+1}C_{i,k+1})e_{i,k+1|k} + K_{i,k+1}\varepsilon_{i,k+1}\\ &\qquad - (\gamma_{i,k+1} - \bar{\gamma}_i)K_{i,k+1}C_{i,k+1}x_{i,k+1} - K_{i,k+1}D_{i,k+1}\\ &\qquad \times v_{k+1})((I - \bar{\gamma}_i K_{i,k+1}C_{i,k+1})e_{i,k+1|k} + K_{i,k+1}\\ &\qquad \times\varepsilon_{i,k+1} - (\gamma_{i,k+1} - \bar{\gamma}_i)K_{i,k+1}C_{i,k+1}x_{i,k+1}\\ &\qquad - K_{i,k+1}D_{i,k+1}v_{k+1})^T\}\\ &\quad\leq (1+e_{i,k+1})(I - \bar{\gamma}_i K_{i,k+1}C_{i,k+1})P_{i,k+1|k}\\ &\qquad\times(I - \bar{\gamma}_i K_{i,k+1}\times C_{i,k+1})^T\\ &\qquad + (1+e_{i,k+1}^{-1})K_{i,k+1}\mathbb{E}\{\varepsilon_{i,k+1}\varepsilon_{i,k+1}^T\}\\ &\qquad\times K_{i,k+1}^T + \tilde{\gamma}_i K_{i,k+1}C_{i,k+1}\mathbb{E}\{x_{i,k+1}x_{i,k+1}^T\}\\ &\qquad\times C_{i,k+1}^T K_{i,k+1}^T - K_{i,k+1}\mathbb{E}\{\varepsilon_{i,k+1}v_{k+1}^T\}D_{i,k+1}^T\\ &\qquad\times K_{i,k+1}^T - K_{i,k+1}D_{i,k+1}\mathbb{E}\{v_{k+1}\varepsilon_{i,k+1}^T\}K_{i,k+1}^T\\ &\qquad + K_{i,k+1}D_{i,k+1}Q_{k+1}D_{i,k+1}^T K_{i,k+1}.\end{aligned} \tag{19}$$

Noting the fact

$$\varepsilon_{i,k+1}\varepsilon_{i,k+1}^T \leq \varepsilon_{i,k+1}^T\varepsilon_{i,k+1}I \tag{20}$$

we have

$$\mathbb{E}\{\varepsilon_{i,k+1}\varepsilon_{i,k+1}^T\} \leq \left((1+\theta_i)\bar{Y}_{i,k+1}/\theta_i^2 + \left(1+\theta_i{}^{-1}\right)\sigma_i^2\right)I. \tag{21}$$

By using Lemma 3 again, the term $\mathbb{E}\{x_{i,k+1}x_{i,k+1}^T\}$ can be derived as follows:

$$\begin{aligned}&\mathbb{E}\{x_{i,k+1}x_{i,k+1}^T\}\\ &\quad= \mathbb{E}\{(e_{i,k+1|k} + \hat{x}_{i,k+1|k})(e_{i,k+1|k} + \hat{x}_{i,k+1|k})^T\}\\ &\quad\leq (1+g_{i,k+1})P_{i,k+1|k} + \left(1+g_{i,k+1}^{-1}\right)\hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^T.\end{aligned} \tag{22}$$

According to

$$\begin{aligned}\varepsilon_{i,k+1} &= y_{i,k+1} - y_{i,t_l^i}\\ &= \gamma_{i,k+1}C_{i,k+1}x_{i,k+1} + D_{i,k+1}v_{k+1} - y_{i,t_l^i}\end{aligned} \tag{23}$$

for $k+1 \in [t_l^i, t_{l+1}^i)$ $(l \geq 0)$, we have

$$\begin{aligned}&\mathbb{E}\{\varepsilon_{i,k+1}v_{k+1}^T\}\\ &\quad= \mathbb{E}\left\{\left(\gamma_{i,k+1}C_{i,k+1}x_{i,k+1} + D_{i,k+1}v_{k+1} - y_{i,t_l^i}\right)v_{k+1}^T\right\}\\ &\quad= \rho_{i,k+1}D_{i,k+1}Q_{k+1}.\end{aligned} \tag{24}$$

Combining (21) and (22) together with (24), we have

$$\begin{aligned}&P_{i,k+1|k+1}\\ &\quad\leq (1+e_{i,k+1})(I - \bar{\gamma}_i K_{i,k+1}C_{i,k+1})P_{i,k+1|k}\\ &\qquad\times(I - \bar{\gamma}_i\times K_{i,k+1}C_{i,k+1})^T + \left(1+e_{i,k+1}^{-1}\right)\\ &\qquad\times\left(\left(1+\theta_i\right)\bar{Y}_{i,k+1}/\theta_i^2 + (1+\theta_i{}^{-1})\sigma_i^2\right)K_{i,k+1}K_{i,k+1}^T\\ &\qquad + \tilde{\gamma}_i(1+g_{i,k+1})\times K_{i,k+1}C_{i,k+1}P_{i,k+1|k}C_{i,k+1}^T K_{i,k+1}^T + \tilde{\gamma}_i\\ &\qquad\times\left(1+g_{i,k+1}^{-1}\right)K_{i,k+1}C_{i,k+1}\hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^T C_{i,k+1}^T\\ &\qquad\times K_{i,k+1}^T + (1-2\rho_{i,k+1})K_{i,k+1}D_{i,k+1}Q_{k+1}\\ &\qquad\times D_{i,k+1}^T K_{i,k+1}^T.\end{aligned} \tag{25}$$

Then, it is easily obtained from (18) that $P_{i,k+1|k+1} \leq \Pi_{i,k+1|k+1}$ and now the proof is complete. ■

In the following theorem, the filter gain is parameterized by minimizing the upper bound derived in Theorem 1.

Theorem 2: For all $1 \leq i \leq n$, suppose that $\lambda_i\theta_i \geq 1$. The upper bound on the filtering error covariance can be minimized with the following gain parameter:

$$K_{i,k+1} = (1 + e_{i,k+1})\bar{\gamma}_i \Pi_{i,k+1|k} C_{i,k+1}^T \Theta_{i,k+1}^{-1} \tag{26}$$

where

$$\begin{aligned}\Theta_{i,k+1} \triangleq\ & (1 + e_{i,k+1})\bar{\gamma}_i^2 C_{i,k+1}\Pi_{i,k+1|k}C_{i,k+1}^T \\ & + \left(1 + e_{i,k+1}^{-1}\right)\left((1+\theta_i)\bar{Y}_{i,k+1}/\theta_i^2 + \left(1 + \theta_i{}^{-1}\right) \times \sigma_i^2\right)I \\ & + \tilde{\gamma}_i(1 + g_{i,k+1})C_{i,k+1}\Pi_{i,k+1|k}C_{i,k+1}^T \\ & + \tilde{\gamma}_i\left(1 + g_{i,k+1}^{-1}\right)C_{i,k+1}\hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^T C_{i,k+1}^T \\ & + (1 - 2\rho_{i,k+1})D_{i,k+1}Q_{k+1}D_{i,k+1}^T.\end{aligned}$$

Proof: The trace of the matrix $\Pi_{i,k+1|k+1}$ can be manipulated as follows:

$$\begin{aligned}&\mathrm{tr}\{\Pi_{i,k+1|k+1}\} \\ &\quad = (1 + e_{i,k+1})\mathrm{tr}\big\{(I - \bar{\gamma}_i K_{i,k+1}C_{i,k+1})\Pi_{i,k+1|k} \\ &\qquad\qquad \times (I - \bar{\gamma}_i \times K_{i,k+1}C_{i,k+1})^T\big\} + (1 + e_{i,k+1}^{-1}) \\ &\qquad \times \left((1+\theta_i)\bar{Y}_{i,k+1}/\theta_i^2 + \left(1 + \theta_i{}^{-1}\right)\sigma_i^2\right)\mathrm{tr}\big\{K_{i,k+1}K_{i,k+1}^T\big\} \\ &\qquad + \tilde{\gamma}_i(1 + g_{i,k+1}) \times \mathrm{tr}\big\{K_{i,k+1}C_{i,k+1}\Pi_{i,k+1|k}C_{i,k+1}^T K_{i,k+1}^T\big\} \\ &\qquad + \tilde{\gamma}_i\left(1 + g_{i,k+1}^{-1}\right) \\ &\qquad \times \mathrm{tr}\big\{K_{i,k+1}C_{i,k+1}\hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^T C_{i,k+1}^T K_{i,k+1}^T\big\} \\ &\qquad + (1 - 2\rho_{i,k+1})\mathrm{tr}\big\{K_{i,k+1}D_{i,k+1}Q_{k+1}D_{i,k+1}^T K_{i,k+1}^T\big\}.\end{aligned} \tag{27}$$

Then, taking partial derivative of $\mathrm{tr}\{\Pi_{i,k+1|k+1}\}$ with respect to $K_{i,k+1}$ leads to

$$\begin{aligned}&\frac{\partial \mathrm{tr}\{\Pi_{i,k+1|k+1}\}}{\partial K_{i,k+1}} \\ &\quad = -2(1 + e_{i,k+1})\bar{\gamma}_i\Pi_{i,k+1|k}C_{i,k+1}^T \\ &\qquad + 2(1 + e_{i,k+1})\bar{\gamma}_i^2 K_{i,k+1}C_{i,k+1}\Pi_{i,k+1|k}C_{i,k+1}^T \\ &\qquad + 2\left(1 + e_{i,k+1}^{-1}\right) \\ &\qquad \times \left((1+\theta_i)\bar{Y}_{i,k+1}/\theta_i^2 + \left(1 + \theta_i{}^{-1}\right) \times \sigma_i^2\right)K_{i,k+1} \\ &\qquad + 2\tilde{\gamma}_i(1 + g_{i,k+1})K_{i,k+1}C_{i,k+1} \\ &\qquad \times \Pi_{i,k+1|k}C_{i,k+1}^T + 2\tilde{\gamma}_i\left(1 + g_{i,k+1}^{-1}\right)K_{i,k+1}C_{i,k+1} \\ &\qquad \times \hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^T C_{i,k+1}^T + 2(1 - 2\rho_{i,k+1})K_{i,k+1} \\ &\qquad \times D_{i,k+1}Q_{k+1}D_{i,k+1}^T.\end{aligned} \tag{28}$$

By setting $(\partial \mathrm{tr}\{\Pi_{i,k+1|k+1}\}/\partial K_{i,k+1}) = 0$, the gain parameter $K_{i,k+1}$ can be determined as follows:

$$K_{i,k+1} = (1 + e_{i,k+1})\bar{\gamma}_i\Pi_{i,k+1|k}C_{i,k+1}^T\Theta_{i,k+1}^{-1}. \tag{29}$$

Therefore, the proof of this theorem is complete now. ■

Based on the above-established results, an upper bound on the filtering error covariance is found in terms of two sets of recursions and, subsequently, the desired filters are designed so as to minimize the obtained upper bound at each time instant.

Remark 6: In this article, our focus is on the recursive filter design issue for a class of time-varying CNs over a finite time-horizon. Note that, since the effects of the nonlinearity and the dynamic ETM have been taken into account in the filter design, it is impossible to obtain the precise expression of the filtering error covariance. As such, we have chosen an alternative way, i.e., derive an upper bound of filtering error covariance and then minimize such a bound by properly selecting the filter parameters at each time step. It is worth mentioning that the boundedness of the upper bound can be naturally ensured in the case of finite time-horizon.

Remark 7: Until now, the recursive filtering problem has been solved for a class of time-varying CNs subject to switching topologies and random sensor failures. In Theorem 1, an upper bound on the filtering error covariance has been obtained for each network node. Then, the obtained upper bound has been minimized at each time step by parameterizing the desired filter gain in Theorem 2. Furthermore, it is notable that, by letting $\theta_i \to \infty$, our main results derived in Theorems 1 and 2 under the dynamic ETMs can be specialized to the special case of static ETMs.

IV. Illustrative Examples

In this section, a numerical simulation example is given to show the effectiveness of the proposed filtering algorithm.

Consider the CN (1) consisting of three nodes over the finite horizon $k \in [0, 60]$ with the following nonlinear function:

$$f_k(x_{i,k}) = E_k x_{i,k} + F_k(x_{i,k})$$

where

$$E_k = \begin{bmatrix} 0.1 & 0.01 + 0.01\cos(k) \\ 0.15 & 0.2 \end{bmatrix}, \quad F_k(x_{i,k}) = 0.1\sin(x_{i,k})$$

and let the probabilities be $p_{11} = 0.2$ and $p_{21} = 0.4$.

The outer coupling matrix subject to π_k are selected as

$$W_1 = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & -0.2 \end{bmatrix}$$
$$W_2 = \begin{bmatrix} -0.1 & 0.1 & 0.0 \\ 0.1 & -0.2 & 0.1 \\ 0 & 0.1 & -0.1 \end{bmatrix}$$

and the other parameters of (1) are given as follows: $\Gamma = \mathrm{diag}\{1, 1\}$, $B_{1,k} = \begin{bmatrix}0.2 & 0.18\end{bmatrix}^T$, $B_{2,k} = \begin{bmatrix}0.15 & 0.1\end{bmatrix}^T$, and $B_{3,k} = \begin{bmatrix}0.3 & 0.12\end{bmatrix}^T$.

The parameters of the measurement model (3) are given as follows: $C_{1,k} = [1.2 \;\; 0.6{+}0.05\cos(k)]$, $C_{2,k} = [1{+}0.05\sin(k) \;\; 0.9]$, $C_{3,k} = [0.8 + 0.05\sin(k) \;\; 0.6 + 0.05\cos(k)]$, $D_{1,k} = 0.15$, $D_{2,k} = 0.3$, $D_{3,k} = 0.2$, and the random sensor failure rates $\gamma_{i,k}$ $(i = 1, 2, 3)$ are assumed to obey the following probability density function:

$$p(s_i) = \begin{cases} 0, & s_i = 0 \\ 0.2, & s_i = 0.5 \\ 0.8, & s_i = 1 \end{cases}$$

which infers $\bar{\gamma}_i = 0.9$ and $\tilde{\gamma}_i = 0.04$.

Considering the dynamic triggering condition (4), (5), the thresholds are given by $\sigma_1 = \sigma_2 = \sigma_3 = 0.3$. In addition, we choose $\lambda_1 = \lambda_2 = \lambda_3 = 0.2$ and $\theta_1 = \theta_2 = \theta_3 = 10$ to satisfy Lemma 1. The covariances of the process noise ω_k and the measurement noise υ_k are $R_k = 1$ and $Q_k = 1$, respectively. The initial value of the state $x_{i,0}$ follows the zero-mean Gaussian distribution with the covariance $P_{i,0} = \mathrm{diag}\{1, 1\}$ and the initial value of the internal dynamic variable is taken as $\eta_0^1 = \eta_0^2 = \eta_0^3 = 1$. Moreover, the introduced parameters are selected as $a_{i,k} = b_{i,k} = c_{i,k} = d_{i,k} = e_{i,k} = g_{i,k} = 1$. According to (26) in Theorem 2, the filter parameters $K_{i,k+1}$ for $i = 1, 2, 3$ can be calculated at each time step.

Simulation results are shown in Figs. 1–5. Figs. 1–3 show the state trajectories and their estimates for state $x_{i,k}$ $(i = 1, 2, 3)$, respectively. Fig. 4 shows the trace of the minimal upper bound $\Pi_{i,k|k}$ and the mean square error (MSE) for the estimation of the state defined by

$$\mathrm{MSE}_{i,k} \triangleq \frac{1}{M}\sum_{t=1}^{M}\sum_{s=1}^{2}\left(x_{i,k}^s - \hat{x}_{i,k|k}^s\right)^2 \tag{30}$$

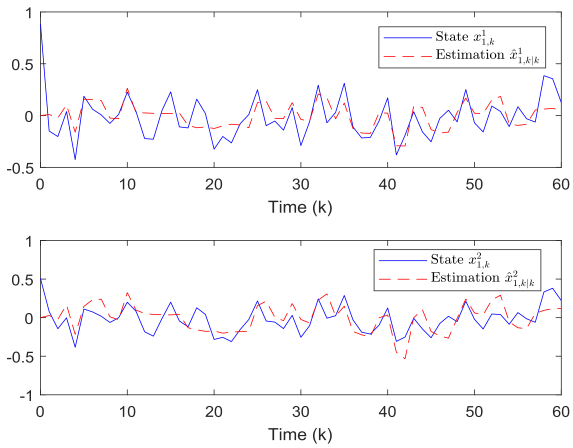

Fig. 1. State x_1 and its estimate.

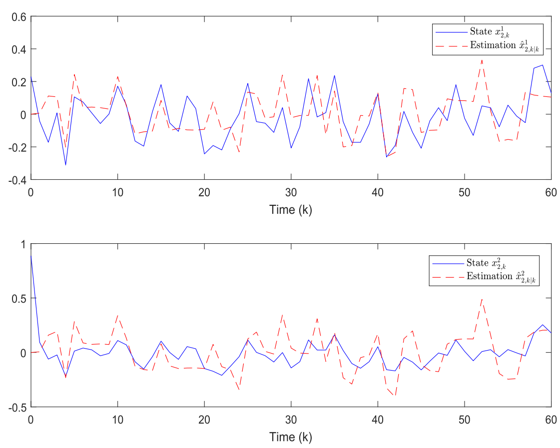

Fig. 2. State x_2 and its estimate.

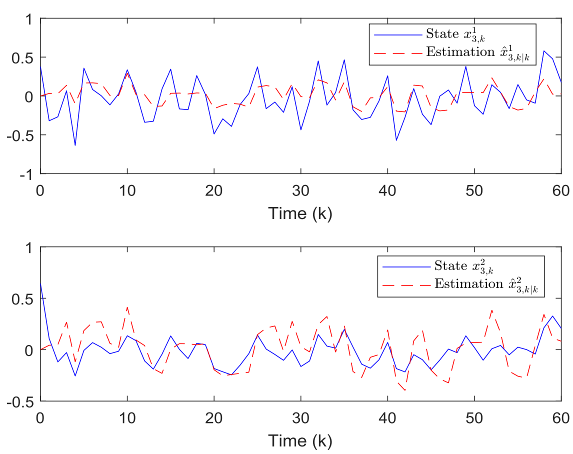

Fig. 3. State x_3 and its estimate.

where $M = 300$ is the implementation number of the independent experiments. The triggering instants of each node under the dynamic ETM can be seen in Fig. 5. The simulation results have illustrated the validity of the dynamic event-triggered filtering algorithm proposed in this article.

For node i $(i = 1, 2, 3)$, we define the triggering rate $\mathcal{J}_i$ as the transmission performance level by $\mathcal{J}_i \triangleq \mathcal{N}_i/N$, where $\mathcal{N}_i$ denotes the number of actually transmitted data and $N = 60$ is the length

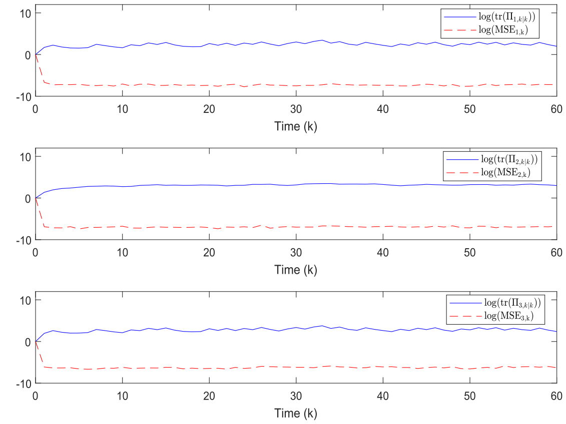

Fig. 4. Trace of error variance and its upper bound for nodes 1, 2, and 3.

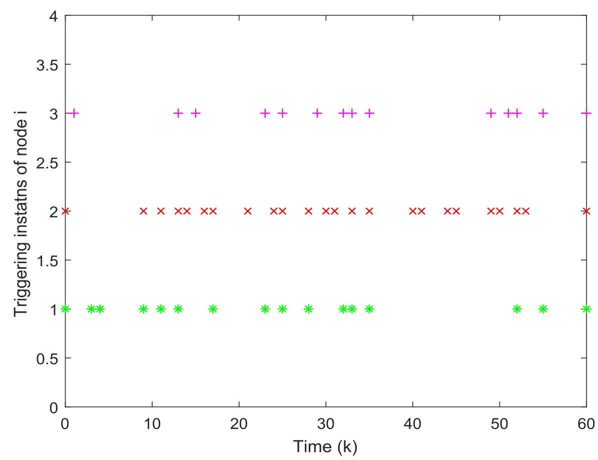

Fig. 5. Triggering instants.

TABLE I
TRIGGERING RATES FOR NODE i $(i = 1, 2, 3)$

	Node 1		Node 2		Node 3	
	$\mathcal{N}_1$	$\mathcal{J}_1$	$\mathcal{N}_2$	$\mathcal{J}_2$	$\mathcal{N}_3$	$\mathcal{J}_3$
Dynamic ETM ($\theta_i = 10$)	16	26.7%	24	40%	14	23.3%
Dynamic ETM ($\theta_i = 20$)	19	31.6%	26	43.3%	15	25%
Dynamic ETM ($\theta_i = 80$)	22	36.7%	27	45%	16	26.7%
Static event-triggering case	23	38.3%	29	48.3%	18	30%

of finite time-horizon. In order to see the relationship between the triggering rate and parameter θ_i (including the special case $\theta_i \to +\infty$), the triggering rate is presented in Table I with different values of θ_i. It appears that the triggering rate monotonically increases when θ_i increases, and the triggering rate is maximum under the static ETM. Therefore, we can conclude that the dynamic ETM is more efficient than the static ETM in reducing transmission, and thus has more potential to save the communication cost.

On the other hand, we provide Fig. 6 to illustrate the effects of the triggering parameter θ_i on estimation performance. It is observed that the traces of the upper bound on the filtering error covariance decrease monotonically as θ_i increases, and the upper bound is minimum when $\theta_i \to +\infty$. Therefore, the dynamic ETM degrades a bit the estimation performance when comparing to the static ETM. From Table I and Fig. 6, it can be concluded that the proposed dynamic ETM reduces the communication burden at the cost of sacrificing certain filtering performance of CNs. Nevertheless, all the simulation results have

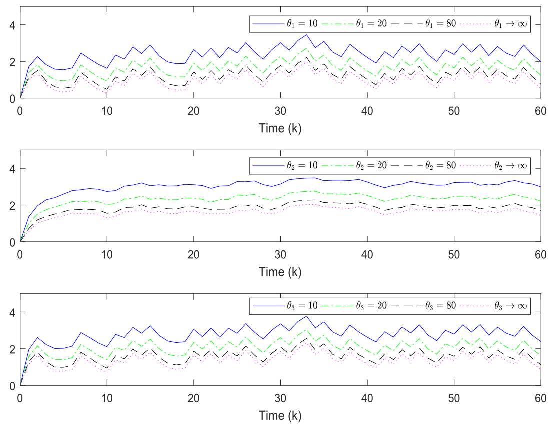

Fig. 6. Traces of the minimal upper bounds for node i ($i = 1, 2, 3$) with different values of θ_i.

shown the effectiveness of the proposed recursive filtering approach under dynamic ETMs.

V. Conclusion

In this article, we have discussed the recursive filtering issue for a class of CNs with switching topologies, random sensor failures, and dynamic ETMs. The switching topology of the network under consideration has been governed by a Markov chain, and the sensor failures have been assumed to occur in a random way. Furthermore, a dynamic event-triggered transmission strategy has been adopted in order to save energy. By employing the induction method, an upper bound on the filtering error covariance for each node has been obtained, which has been minimized subsequently by properly designing the filter parameters. Finally, through a simulation example, we have shown the effectiveness of the recursive filtering approach proposed. One of our future research topics would be to extend the main results obtained in this article to more complex systems [25], [38].

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